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Results on pine tree height increment using the Gieruszyński and Meixner formulas

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SUMMARY

This paper presents the results of investigations into the effect of the age of trees and length of growth period on the accuracy of determination of periodical tree height increments using ten formulas. The experimental material comprised the results of analyses of 200 pine-tree stems which were selected from eight consecutive sub-age classes, from IIa to Vb. All stands from which sample trees were collected were growing in conditions of fresh mixed coniferous forest. The same calendar growth period (1989–1999) was adopted for each tree, in order to eliminate the effect of additional factors such as site, climate and meteorology on the height increment.

Key words: height increment, age of trees, length of growth period, analysis of variance, accuracy of methods, Scots pine

1. Introduction

Periodical height increments of trees are considered a major dendrometric characteristic, since they exert a direct influence on the accuracy of determination of the tree and stand volume increment. It is a variable characteristic and depends on several factors (Beker, 1997; Kaźmierczak, 2004, 2005; Lemke, 1972a, 1972b; Najgrakowski, 1998; Rymer-Dudzińska, 1997, 1998). The most important of these factors include the species of the tree, its age, biosocial position, soil and climatic-meteorological conditions. A direct evaluation of the height increments of standing trees is quite difficult, and sometimes simply impossible.

Gieruszyński (1961) and Meixner (1977) proposed indirect methods of determining height increments with the use of the following formulas:

- Gieruszyński (G) $$Zh = \frac{H}{D_{zk}} \cdot Zd \qquad (1)$$

- Meixner 1 (M1) $$Zh = \frac{H}{D_{zk} - K} \cdot Zd = \frac{H}{D_{bk}} \cdot Zd \qquad (2)$$

- Meixner 2 (M2) $$Zh = \frac{H - 1{,}30}{D_{zk}} \cdot Zd \qquad (3)$$

- Meixner 3 (M3) $$Zh = \frac{H - 1{,}30}{D_{zk} - K} \cdot Zd = \frac{H - 1{,}30}{D_{bk}} \cdot Zd \qquad (4)$$

- Meixner 4 (M4) $$Zh = \frac{H}{D_{bk}} \cdot Zd \cdot \left(2 - \frac{Zd}{D_{bk}} \right) \qquad (5)$$

- Meixner 5 (M5) $$Zh = \frac{H}{D_{zk}} \cdot Zd \cdot \left(2 - \frac{Zd}{D_{zk}} \right) \qquad (6)$$

- Meixner 6 (M6) $$Zh = \frac{H}{D_{zk}^2} \cdot Zd \cdot (2 \cdot D_{bk} - Zd) \qquad (7)$$

- Meixner 7 (M7) $$Zh = \frac{H - 1{,}30}{D_{bk}} \cdot Zd \cdot \left(2 - \frac{Zd}{D_{bk}} \right) \qquad (8)$$

- Meixner 8 (M8) $$Zh = \frac{H - 1{,}30}{D_{zk}} \cdot Zd \cdot \left(2 - \frac{Zd}{D_{zk}} \right) \qquad (9)$$

- Meixner 9 (M9) $$Zh = \frac{H - 1{,}30}{D_{zk}^2} \cdot Zd \cdot (2 \cdot D_{bk} - Zd) \qquad (10)$$

where:
Zh = height increment for the considered period,
H = tree height at the end of the growth period,
D_{zk} = breast height diameter outside the bark at the end of the growth period,
D_{bk} = breast height diameter inside the bark at the end of the growth period,
K = d.b.h. bark thickness,
Zd = d.b.h. increment for the examined period.

Investigations into the accuracy of Gieruszyński's formula (1) were conducted by the author himself (Gieruszyński, 1961) as well as by Łapaczewski (1963), Lemke and Meixner (1967), Meixner (1978, 1979, 1981), Drzymała (1997) and Kaźmierczak (2004, 2005). The accuracy of tree height increment determination using the remaining methods (2)–(10) was evaluated by the author himself (Meixner, 1978, 1979, 1981) as well as by Kaźmierczak (2004, 2005) and, for some selected methods, by Drzymała (1997).

The objective of the present study was to evaluate the impact of the age of trees and length of the growth period on the accuracy of calculation of the periodical height increment of single trees using the above methods.

2. Material and methods

The experimental material included selected analysis results for 200 pine-tree stems derived from 8 stands. The following data were used to perform the required analyses: age of sample trees, their breast height diameter, height, and bre ast height diameter and height increments for two growth periods. Mean sample trees were selected following the methodology developed by Draudt. All stands from which the experimental test trees derived were at fresh mixed coniferous forest sites situated in the Zielonka Experimental Forest District. The same calendar growth period extending from 1989 to 1999 was adopted for each tree. The purpose of this assumption was to rule out the effect of additional factors such as site, climate and meteorology on the height increment.

The investigations began with determination of the true Zh value on the basis of the whorls of all trees in the adopted periods Zh_5 (1994–1999) and Zh_{10} (1989–1999). Next, 5- and 10-year height increments were calculated for each tree using Gieruszyński's formula (1) as well as Meixner's nine formulas (2)–(10). Afterwards, the secondary percentage error of the increments calculated in this way was determined, which shows by what percentage of the real increment the increment calculated using a given method was higher or lower.

Adopting the value of the secondary percentage error as a basis, the investigations into the impact of tree age and length of growth period on the accuracy of determination of the height increment by selected methods were represented as a three-factor experiment: 10 formulas x 8 age groups x 2 growth periods. Later, analysis of variance was performed. The analyzed data were presented as a three-dimensional cuboid consisting of 10 lines, 8 columns and 2 layers. Each of the 160 cells contained 25 results.

The analysis of variance was preceded by checking of the compliance of the empirical distribution of secondary percentage errors with the normal distribution. The Shapiro-Wilk test was applied to each method in all age groups and in the two adopted growth periods. In the case of the majority of the examined groups (in 150 out of 160 cases), no basis was found for rejection of the hypothesis of compliance of the secondary percentage error distribution with the normal distribution. This observation made it possible, during further analysis, to apply tests which assumed similarity of the empirical distribution and normal distribution.

The F test for the comparison of many means found significant differences between means. However, this does not answer the question of which group means differ from others. Therefore, once statistically significant differences between means were found to exist, further steps were taken to investigate the nature of these differences, using the procedure of multiple comparisons (Ferguson and Takane, 2002). We used the Tukey test (honestly significant difference), which employs studentized range statistics. The comparisons performed enabled the author to identify internally uniform groups.

3. Results

The results of the three-way analysis of variance are presented in Table 1.

Table 1. Results of three-way analysis of variance

Source of variability		Total square deviation	Degree of freedom	Mean square deviation	F	*p*-level
Intercept		6712674	1	6712674	2387.1	0.000**
Main effect	*formula*	6286758	9	698529	248.4	0.000**
	age group	8351787	7	119312	424.3	0.000**
	period of growth	4772	1	4772	1.7	0.193
Inter-action	*formula x age group*	790636	63	12550	4.5	0.000**
	formula x period of growth	1527	9	170	0.1	0.999
	age group x period of growth	68573	7	9796	3.5	0.001**
	formula x age group x period of growth	4980	63	79	0.03	1.000
	Error	10798207	3840	2812		

** - statistically significant influence at level 0.01

The following conclusions can be drawn on the basis of these results:

1. Main effect: *formulas* – statistically significant *p*-value indicates that the considered formulas have different accuracy.
2. Main effect: *age group* – statistically significant *p*-value indicates that the age of the investigated trees affects the accuracy of determination of the periodical height increment.
3. Main effect: *length of growth period* – statistically insignificant *p*-value – the length of the growth period (5 or 10 years) exerts no influence on the error in determination of the height increment.
4. Two-way interaction – *formulas and age* – statistically significant *p*-value indicates interaction between formulas and age, in other words, the method and age jointly influence the accuracy of determination of the height increment.
5. Two-way interaction – *formulas and length of growth period* – statistically insignificant *p*-value indicates absence of interaction.
6. Two-way interaction – *age and length of growth period* – *p*-value statistically significant but caused by the important role of age in determining the height increment and the lack of influence of the length of the growth period.
7. Three-way interaction: *formulas and age and length of the growth period* – statistically insignificant *p*-value indicates that the determination error of the height increment depends on the applied formula for a specific tree age, but it does not depend on the length of the considered growth period.

Further detailed assessment of differences between the accuracy of the considered methods and the impact of age as well as the length of the growth period was carried out on the basis of multiple comparison procedures using the Tukey method. This made it possible to identify internally uniform groups, and the results of these comparisons are presented in Tables 2–3. All cases are arranged in order, from the highest mean negative to the highest mean positive. Additionally, the impact of the considered factors on the results is illustrated. Figures 1–4 present the mean values and the 95% confidence intervals determined for them.

These can be interpreted as follows:

1. With the Tukey test, three internally uniform groups of *formulas* based on accuracy of height increment determination (Table 2). Gieruszyński's

formula and Meixner's first three formulas do not differ significantly from one another, but they differ from the remaining formulas (Table 2). According to the Tukey test, the second uniform group comprises Meixner's 5th and 9th formulas, whereas Meixner's 4th formula differs significantly from the two remaining ones and forms a separate group (Table 2).

2. The majority of the examined *age groups* form specific, separate uniform groups (Table 3) with the exception of 24- and 33-year-old trees, which form one group.

Table 2. Multiple comparison Tukey's test with respect to the formula for calculating the height increment

Formula	Mean	Homogeneous groups, α=0.05						
		1	2	3	4	5	6	7
M 2	-12.91	a						
G	-7.27	a	b					
M 3	-2.74	a	b					
M 1	3.56		b					
M 9	48.02			c				
M 6	57.59			c	d			
M 8	66.00				d	e		
M 5	76.73					e	f	
M 7	84.38						f	
M 4	96.29							g

Table 3. Multiple comparison Tukey's test with respect to age group of trees

Age group of trees (years)	Mean	Homogeneous groups, α=0.05						
		1	2	3	4	5	6	7
24	-24.07	a						
33	-14.81	a						
43	7.43		b					
55	32.00			c				
62	54.38				d			
72	75.29					e		
84	90.98						f	
92	106.53							g

3. The *length of the growth period* did not affect significantly the accuracy of determination of the height increment, therefore both increment groups were included in the same uniform group (5 years : 39.87; 10 years : 42.06).

4. Figure 1 shows the *formula x age group* interaction. A division of methods into two categories is apparent. The first category comprises Gieruszyński's and Meixner's first three methods. The second category contains the remaining methods, i.e. Meixner's 4th to 9th. The highest mean negative errors were obtained using Meixner's 2nd method, while the highest positive errors came from Meixner's 4th method. Methods in the first category underestimate tree height increment up to an age of 60 years, while later the result of this determination is exaggerated. Methods in the second category slightly underestimate the height increment in the youngest trees.

However they overestimate the measurement in older trees, and the value of the mean error increases with age.

5. The effect of the *formula x length of growth period* interaction is shown in Figure 2. The parallel profiles for concurrent effects of the two factors indicates lack of interaction. This occurs in two groups of formulas. The first group includes methods 1–4, and the second methods 5–10. This also corroborates the correctness of the splitting of the methods into two categories as identified earlier.
6. The *age group x length of growth period* interaction is shown in Figure 3. Both profiles cross several times for older trees, from approximately 50 years of age upwards; there is significant interaction in the analysis of variance.
7. The effect of *formula x age group x length of growth period* interaction is shown in Figure 4. Profiles are seen to be almost identical in both growth periods; there is no significant interaction in the analysis of variance. Also here the profiles run parallel in the two groups of formulas identified earlier, confirming the appropriateness of the division of the considered height increment determination methods into two separate categories.

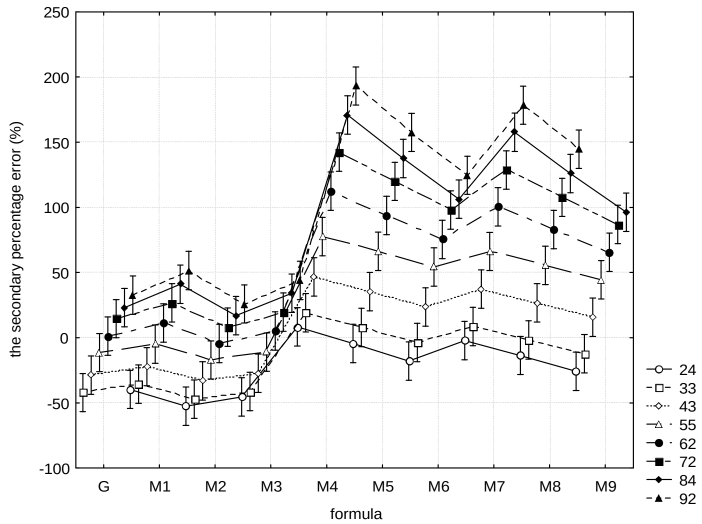

Figure 1. Group profile based on formula and age of trees

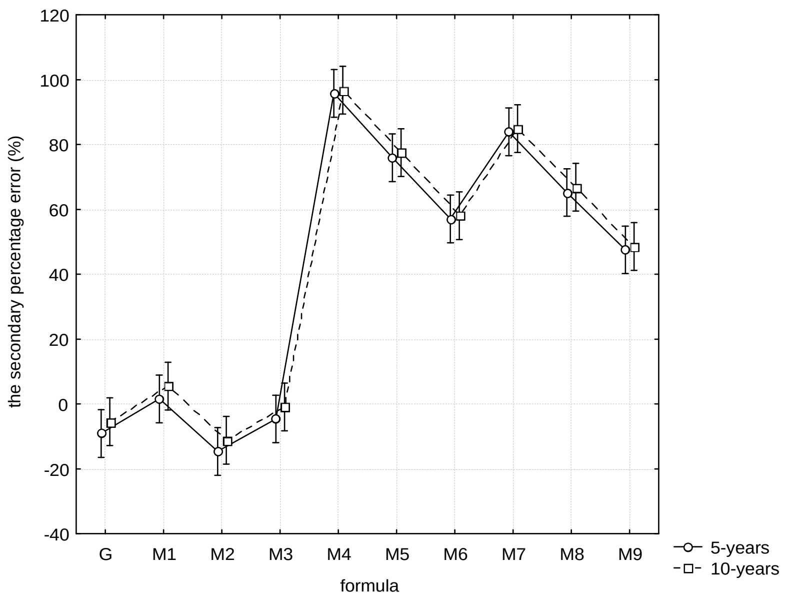

Figure 2. Group profile based on formula and period of growth

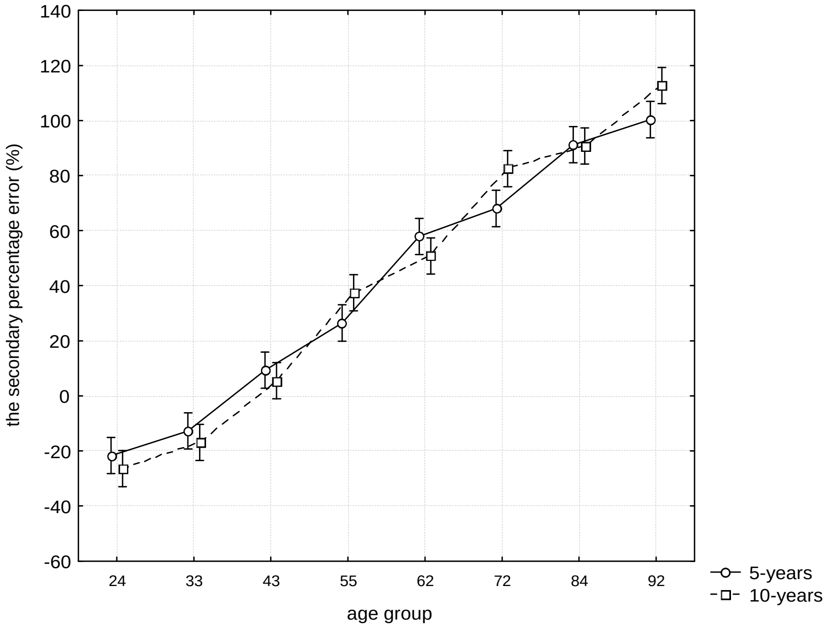

Figure 3. Group profile based on age of trees and period of growth

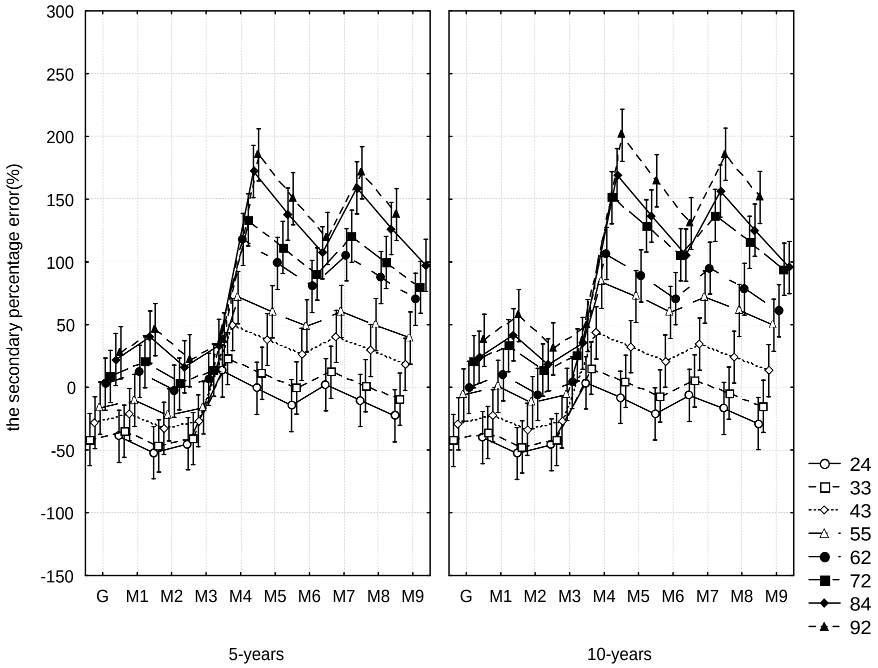

Figure 4. Group profile based on formula, age of trees and period of growth

Recapitulation of the analysis of variance and multiple comparisons led to conclusions which coincided with the results of earlier investigations by the author (Kaźmierczak, 2004, 2005).

4. Conclusions

1. The accuracy of determination of height increment depends on the applied formula and on the age of the trees.
2. No significant influence of length of growth period on the accuracy of height increment determination has been found.
3. Based on the accuracy of the results obtained, the methods used to determine the height increment of trees can be divided into two groups. The first group comprises Gieruszyński's method and Meixner's first three methods, while the second group consists of the remaining formulas.

4. Methods in the first group underestimate very significantly the value of the calculated height increments in trees up to about 50 years of age, while in trees older than 70 years they overestimate the value.
5. Methods in the second category underestimate the height increment of young trees only slightly. Later the results carry high positive errors, which increase with the age of the trees.
6. Formulas in the second category are better suited to calculate tree height increments in young trees, while those in the first group should be used to determined this parameter in older trees.

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